Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Five students at a meeting remove their name tags and put them in a hat; the five students then each randomly choose one of the name tags from the bag. What is the probability that exactly one person gets their own name tag?

2. Compute
$$\sum_{n=1}^{\infty} \frac{(7n+32)\cdot 3^n}{n\cdot(n+2)\cdot 4^n}.$$

3. Find the unique polynomial $P(x)$ with coefficients taken from the set $\{-1,0,1\}$ and with least possible degree such that $P(2010)\equiv 1 \pmod 3$, $P(2011)\equiv 0 \pmod 3$, and $P(2012)\equiv 0 \pmod 3$.

4. Let T_n denote the number of terms in $(x+y+z)^n$ when simplified, i.e. expanded and like terms collected, for non-negative integers $n\geq 0$. Find
$$\sum_{k=0}^{2010}(-1)^k T_k = T_0 - T_1 + T_2 - \cdots - T_{2009} + T_{2010}.$$

5. Two ants begin on opposite corners of a cube. On each move, they can travel along an edge to an adjacent vertex. Find the probability they both return to their starting position after 4 moves.

6. An unfair coin has a $2/3$ probability of landing on heads. If the coin is flipped 50 times, what is the probability that the total number of heads is even?

7. Compute the sum of all n for which the equation $2x+3y=n$ has exactly 2011 nonnegative ($x,y\geq 0$) integer solutions.

8. Let $\{a_i\}_{i=1,2,3,4}, \{b_i\}_{i=1,2,3,4}, \{c_i\}_{i=1,2,3,4}$ be permutations of $\{1,2,3,4\}$. Find the minimum of $a_1b_1c_1 + a_2b_2c_2 + a_3b_3c_3 + a_4b_4c_4$.

9. How many functions f that take $\{1,2,3,4,5\}$ to $\{1,2,3,4,5\}$, not necessarily injective or surjective (i.e. one-to-one or onto), satisfy $f(f(f(x))) = f(f(x))$ for all x in $\{1,2,3,4,5\}$?

10. Find the number of ways of filling a $2\times 2\times 8$ box with 16 $1\times 1\times 2$ boxes (rotations and reflections of the $2\times 2\times 8$ box are considered distinct).