Time limit: 110 minutes.
Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. An isosceles triangle has two of its three angles measuring 30° and x°. What is the sum of the possible values of x?

2. An ant starts at the point $(1, 1)$. It can travel along the integer lattice, only moving in the positive x and y directions. What is the number of ways it can reach $(5, 5)$ without passing through $(3, 3)$?

3. A rectangular pool has diagonal 17 *units* and area 120 $units^2$. What is the perimeter of the pool?

4. Lily draws a square of side length s and gives it to the 3D-fier, which turns it into a cube. Kelly gives the 3D-fier a circle with radius r and receives a cylinder of volume 5 times the volume of the cube. If the 3D-fier always extends the 2D object the same length, what is r/s?

5. Carla has 100 stacks of pennies. The stacks have 1 penny, 2 pennies, 3 pennies, up to 100 pennies. Carla makes a move by adding one penny to each of any 99 stacks. What is the least number of moves Carla can make such that all 100 stacks have the same amount of pennies?

6. How many natural numbers less than 2021 are coprime to 2021?

7. Square $ABCD$ has side length 3. Point E lies on line BC, outside of segment BC and closer to C. A semicircle is drawn with diameter AE. The perpendicular from B to AE has length $\frac{12}{5}$. What is the area of the semicircle?

8. What is the least positive integer n such that 2020! is not a multiple of 7^n?

9. How many ways are there to color every square of an eight-by-eight grid black or white such that for every pair of rows r and s, we have that either $r_i = s_i$ for all $1 \leq i \leq 8$, or $r_i \neq s_i$ for all $1 \leq i \leq 8$?

10. Let ABC be a triangle and D be a point on side BC. Let O be the midpoint of AD. The circle centered at O passing through A intersects AB and AC at E and F (both not A), respectively. If O lies on EF and $\angle ABC$ is five times $\angle ACB$, compute $\angle ABC$ in degrees.

11. Suppose that $f(\frac{1}{x-3}) = \frac{x}{x+1}$ for all $x > 3$. Compute $f(x)$.

12. $\triangle ABC$ has side length $BC = 5$. The angle bisector from A intersects BC at D, with $BD = 2$ and $CD = 3$, and the angle bisector from C intersects AB at F, with $BF = 2$. What is the length of side AC?

13. Imagine that school starts at 8:00 AM. When you drive at 50 mph, you reach 12 minutes early, but when you drive at 40 mph, you reach at 8:15. How many miles do you live from school?

14. In the upcoming pep rally, there will be two dodgeball games, one between the freshmen and sophomores, and the other between the juniors and seniors. Five students have volunteered from each grade. How many ways are there to pick the teams if the only requirements are that there is at least one person on each team and that the teams playing against each other have the same number of people?

15. January 1, 2021 is a Friday. Let p be the probability that a randomly chosen day in the year 2021 is a Friday the 13th given that it is a Friday. Let q be the probability that a randomly chosen day in the year 2021 is a Friday the 13th given that it is the 13th of a month. What is $\frac{p}{q}$?

16. What is the length of the range of x such that $\frac{1}{x} - |\frac{1}{x-1}| > \frac{1}{x-2}$?

17. In $\triangle ABC$, $AB = 2$, $AC = \sqrt{2}$, and $\angle BAC = 105^\circ$. If point D lies on side BC such that $\angle CAD = 90^\circ$, what is the length of BD?

18. 4 couples are sitting in a row. However, two particular couples are fighting, so they are not allowed to sit next to each other. How many ways can these 8 people be seated?

19. A function $f(x)$ is a quadratic with real coefficients. Given $f(0) \neq 0$ and $f(xy+1) = f(x) \cdot f(y) + f(x+y)$ for all $x, y \in \mathbb{R}$, find $f(3)$.

20. If $x^2 + y^2 = 47xy$, then $\log(k(x+y)) = \frac{1}{2}(\log x + \log y)$. Find the value of k.

21. Three spheres are centered at the vertices of a triangle in the horizontal plane and are tangent to each other. The triangle formed by the uppermost points of the spheres has side lengths 10, 26, and $2\sqrt{145}$. What is the area of the triangle whose vertices are at the centers of the spheres?

22. Jane is trying to create a list of all the students of a high school. When she organizes the students into 5, 7, 9, or 13 columns, there are 1, 4, 5, and 10 students left over, respectively. What is the least number of students that could be attending this school?