Time limit: 50 minutes.
Instructions: For this test, you work in teams of eight to solve 15 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Given that the three points where the parabola $y = bx^2 - 2$ intersects the x-axis and y-axis form an equilateral triangle, compute b.

2. Compute the last digit of $2^{\left(3^{\left(4^{\cdots 2014}\right)}\right)}$.

3. A math tournament has a test which contains 10 questions, each of which come from one of three different subjects. The subject of each question is chosen uniformly at random from the three subjects, and independently of the subjects of all the other questions. The test is *unfair* if any one subject appears at least 5 times. Compute the probability that the test is unfair.

4. Let S_n be the sum $S_n = 1 + 11 + 111 + 1111 + \ldots + 111\ldots 11$ where the last number $111\ldots 11$ has exactly n 1's. Find $\lfloor 10^{2017}/S_{2014} \rfloor$.

5. ABC is an equilateral triangle with side length 12. Let O_A be the point inside ABC that is equidistant from B and C and is $\sqrt{3}$ units from A. Define O_B and O_C symmetrically. Find the area of the intersection of triangles O_ABC, AO_BC, and ABO_C.

6. A composition of a natural number n is a way of writing it as a sum of natural numbers, such as $3 = 1 + 2$. Let $P(n)$ denote the sum over all compositions of n of the number of terms in the composition. For example, the compositions of 3 are 3, $1+2$, $2+1$, and $1+1+1$; the first has one term, the second and third have two each, and the last has 3 terms, so $P(3) = 1+2+2+3 = 8$. Compute $P(9)$.

7. Let ABC be a triangle with $AB = 7$, $AC = 8$, and $BC = 9$. Let the angle bisector of A intersect BC at D. Let E be the foot of the perpendicular from C to line AD. Let M be the midpoint of BC. Find ME.

8. Call a function g *lower-approximating* for f on the interval $[a, b]$ if for all $x \in [a, b]$, $f(x) \geq g(x)$. Find the maximum possible value of $\int_1^2 g(x)dx$ where $g(x)$ is a linear lower-approximating function for $f(x) = x^x$ on $[1, 2]$.

9. Determine the smallest positive integer x such that $1.24x$ is the same number as the number obtained by taking the first (leftmost) digit of x and moving it to be the last (rightmost) digit of x.

10. Let a and b be real numbers chosen uniformly and independently at random from the interval $[-10, 10]$. Find the probability that the polynomial $x^5 + ax + b$ has exactly one real root (ignoring multiplicity).

11. Let b be a positive real number, and let a_n be the sequence of real numbers defined by $a_1 = a_2 = a_3 = 1$, and $a_n = a_{n-1} + a_{n-2} + ba_{n-3}$ for all $n > 3$. Find the smallest value of b such that
$$\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{2^n}$$
diverges.

12. Find the smallest L such that

$$\left(1-\frac{1}{a}\right)^b\left(1-\frac{1}{2b}\right)^c\left(1-\frac{1}{3c}\right)^a \leq L$$

for all real numbers a, b, and c greater than 1.

13. Find the number of distinct ways in which $30^{\left(30^{30}\right)}$ can be written in the form $a^{\left(b^c\right)}$, where a, b, and c are integers greater than 1.

14. Convex quadrilateral $ABCD$ has sidelengths $AB = 7$, $BC = 9$, $CD = 15$. A circle with center I lies inside the quadrilateral, and is tangent to all four of its sides. Let M and N be the midpoints of AC and BD, respectively. It can be proven that I always lies on segment MN. If I is in fact the midpoint of MN, find the area of quadrilateral $ABCD$.

15. Marc has a bag containing 10 balls, each with a different color. He draws out two balls uniformly at random and then paints the first ball he drew to match the color of the second ball. Then he places both balls back in the bag. He repeats until all the balls are the same color. Compute the expected number of times Marc has to perform this procedure before all the balls are the same color.