1. If $f(x) = \frac{g(x)}{x}$ where $g(2) = 4$ and $g'(2) = 8$, compute $f'(2)$.

   Answer: 3

   Solution: We use the quotient rule to compute

$$f'(x) = \frac{g'(x) \cdot x - g(x) \cdot 1}{x^2}$$
$$f'(2) = \frac{8 \cdot 2 - 4 \cdot 1}{2^2} = \frac{16-4}{4} = \boxed{3}.$$

2. Compute

$$\lim_{n \to 1} \sum_{k=1}^{25} \frac{n^k - 1}{\ln(n)}.$$

   Answer: 325

   Solution: We begin by applying L'Hopital's Rule to each term in the sum

$$\lim_{n \to 1} \frac{n^k - 1}{\ln(n)} = \lim_{n \to 1} \frac{kn^{k-1}}{1/n} = k.$$

   If we swap the limit and finite sum in our original problem, we can then compute

$$\lim_{n \to 1} \sum_{k=1}^{25} \frac{n^k - 1}{\ln(n)} = \sum_{k=1}^{25} \lim_{n \to 1} \frac{n^k - 1}{\ln(n)} = \sum_{k=1}^{25} k = \frac{(25)(26)}{2} = \boxed{325}.$$

3. Compute

$$\int_0^\pi \sin^{10}(x)\, dx.$$

   Answer: $\frac{63\pi}{256}$

   Solution: We first compute

$$\int_0^\pi \sin^2(x)\, dx = \int_0^\pi \frac{1 - \cos(2x)}{2}\, dx$$
$$= \left(\frac{x}{2} - \frac{\sin(2x)}{4} \right) \Big|_0^\pi = \frac{\pi}{2}.$$

   Now note that $d(-\cos(x)) = \sin(x)$, which allows us to integrate by parts and compute

$$\int_0^\pi \sin^n(x)\, dx = \int_0^\pi \sin^{n-1}(x) d(-\cos(x))\, dx$$
$$= -\sin^{n-1}(x)\cos(x)\big|_0^\pi - \int_0^\pi (n-1)\sin^{n-2}(-\cos^2(x))\, dx$$
$$= 0 + (n-1) \int_0^\pi \sin^{n-2}(1 - \sin^2(x))\, dx$$
$$= (n-1) \int_0^\pi \sin^{n-2}(x)\, dx - (n-1) \int_0^\pi \sin^n(x)\, dx.$$

   Rearranging gives us

$$\int_0^\pi \sin^n(x)\, dx = \frac{n-1}{n} \int_0^\pi \sin^{n-2}(x)\, dx.$$

This gives us a recursive relation between the integrals. From above, when $n = 2$, we have $\int_0^\pi \sin^2(x)\,dx = \frac{\pi}{2}$. Therefore, when $n = 10$, we have

$$\frac{\pi}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8} \cdot \frac{9}{10} = \boxed{\frac{63\pi}{256}}.$$