Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. How many nonnegative integers less than 2019 are not solutions to $x^8 + 4x^6 - x^2 + 3 \equiv 0 \pmod 7$?

2. How many rational numbers can be written in the form $\frac{a}{b}$ such that a and b are relatively prime positive integers and the product of a and b is $(25!)$?

3. Connie finds a whiteboard that has magnet letters spelling $MISSISSIPPI$ on it. She can rearrange the letters, in which identical letters are indistinguishable. If she uses all the letters and does not want to place any Is next to each other, how many distinct rearrangements are possible?

4. In your drawer you have two red socks and a blue sock. You randomly select socks, without replacement, from the drawer. However, every time you take a sock, another blue sock magically appears in the drawer. You stop taking socks when you have a pair of red socks. At this time, say you have x socks total. What is the expected value of x?

5. Let $S(n)$ denote the sum of the digits of positive integers n. For some positive integer k, it is known that $S(k) = 152$ and that $S(k + 1)$ is a multiple of 5. What is the difference between the largest and smallest possible values of $S(k + 1)$?

6. The numbers $1, 2, ..., 13$ are written down, one at a time, in a random order. What is the probability that at no time during this process the sum of all written numbers is divisible by 3?

7. Let $S = 1 + 2 + 3 + ... + 100$. Find $(100!/4!) \mod S$.

8. Let $S_n = \sum_{j=1}^{n} j^3$. Find the smallest positive integer n greater than 100 such that the first three digits of S_n are 100.

9. Edward has a 3×3 tic-tac-toe board and wishes to color the squares using 3 colors. How many ways can he color the board such that there is at least one row whose squares have the same color and at least one column whose squares have the same color? A coloring does not have to contain all three colors and Edward cannot rotate or reflect his board.

10. Let N be a positive integer that is a product of two primes p, q such that $p \leq q$ and for all a, $a^{5N} \equiv a \mod 5N$. Find the sum of p over all possible values N.