1. How many ways are there to choose positive integers x and y such that the lowest common multiple of x and y is 216?

   Answer: 49

   Solution: Note that $216 = 2^3 3^3$. First, consider the power of 2. x must be divisible by 1 of $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, and $2^3 = 8$; similarly, y must be divisible by 1 of these 4 powers of 2. However, we cannot have both x and y be divisible by only 2^0, 2^1, or 2^2 and not 2^3. Hence, there are $4 \cdot 4 - 3 \cdot 3 = 7$ ways of distributing the powers of 2. Similarly, there are 7 ways of distributing the powers of 3. Hence, the number of ways to choose positive integers x and y is $7 \cdot 7 = \boxed{49}$.

2. Consider tangent circles γ_1 and γ_2 with centers O_1, O_2 and radii R, r with $r < R$, respectively. Let $\overline{AB}$ be a common external tangent of length 16. The area of ABO_1O_2 is 160. Find the ordered pair (r, R).

   Answer: (4,16)

   Solution: By the area of a trapezoid, $160 = \frac{16\times(r+R)}{2}$. So, $r + R = 20$. Also, Pythagorean Theorem gives us $(R - r)^2 + 16^2 = (R + r)^2$. So, $R - r = 12$. Solving gives the answer.

3. Consider the set of odd integers $S = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21\}$. Let $\wp(S)$ denote the set of subsets of S. Given $T \in \wp(S)$, we define to α_T to be the sum of the elements of T. Compute $\sum_{T\in\wp(S)} \alpha_T$.

   Answer: 123904

   Solution: Each number in S appears in the same number of subsets: namely, $2^{10} = 1024$ of them. Then the final answer is 1024 times the sum of the elements of S. Conveniently, the sum of the first n odd numbers is n^2 so the sum of S is $11^2 = 121$. The final answer is then $121 \times 1024 = \boxed{123904}$. Pro tip: it's easiest to do this computation by multiplying 1024 by 11 twice, since multiplying by 11 is pretty easy.

   More generally, if we replace S with the set of the first n odd numbers, the answer will be $2^{n-1}n^2$.