Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. A circle with radius 1 is circumscribed by a rhombus. What is the minimum possible area of this rhombus?

2. Let $\triangle ABC$ be a right triangle with $\angle ABC = 90°$. Let the circle with diameter BC intersect AC at D. Let the tangent to this circle at D intersect AB at E. What is the value of $\frac{AE}{BE}$?

3. Square $ABCD$ has side length 4. Points P and Q are located on sides BC and CD, respectively, such that $BP = DQ = 1$. Let AQ intersect DP at point X. Compute the area of triangle PQX.

4. Let $ABCD$ be a quadrilateral such that $AB = BC = 13$, $CD = DA = 15$ and $AC = 24$. Let the midpoint of AC be E. What is the area of the quadrilateral formed by connecting the incenters of ABE, BCE, CDE, and DAE?

5. Find the smallest possible number of edges in a convex polyhedron that has an odd number of edges in total has an even number of edges on each face.

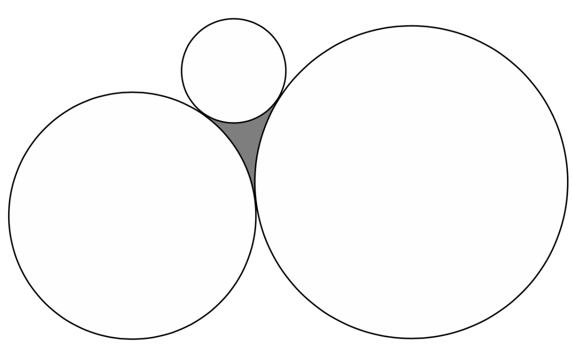
6. Consider triangle ABC on the coordinate plane with $A = (2, 3)$ and $C = (\frac{96}{13}, \frac{207}{13})$. Let B be the point with the smallest possible y-coordinate such that $AB = 13$ and $BC = 15$. Compute the coordinates of the incenter of triangle ABC.

7. Let ABC be an acute triangle with $BC = 4$ and $AC = 5$. Let D be the midpoint of BC, E be the foot of the altitude from B to AC, and F be the intersection of the angle bisector of $\angle BCA$ with segment AB. Given that AD, BE, and CF meet at a single point P, compute the area of triangle ABC. Express your answer as a common fraction in simplest radical form.

8. Consider an acute angled triangle $\triangle ABC$ with side lengths 7, 8, and 9. Let H be the orthocenter of ABC. Let Γ_A, Γ_B, and Γ_C be the circumcircles of $\triangle BCH$, $\triangle CAH$, and $\triangle ABH$ respectively. Find the area of the region $\Gamma_A \cup \Gamma_B \cup \Gamma_C$ (the set of all points contained in at least one of Γ_A, Γ_B, and Γ_C).

9. Let ABC be a right triangle with hypotenuse AC. Let G be the centroid of this triangle and suppose that we have $AG^2 + BG^2 + CG^2 = 156$. Find AC^2.

10. Three circles with radii 23, 46, and 69 are tangent to each other as shown in the figure below (figure is not drawn to scale).



Find the radius of the largest circle that can fit inside the shaded region.