1. If x, y, and z are real numbers such that $x^2 + 2y^2 + 3z^2 = 96$, what is the maximum possible value of $x + 2y + 3z$?

   Answer: 24

   Solution: By the Cauchy-Schwarz inequality, we have

$$\begin{aligned}
&(1+2+3)(x^2+2y^2+3z^2) \geq (x+2y+3z)^2 \\
\Rightarrow\ &6 \cdot 96 \geq (x+2y+3z)^2 \\
\Rightarrow\ &24 \geq x+2y+3z \geq -24 \\
\Rightarrow\ &\boxed{24} \geq x+2y+3z.
\end{aligned}$$

2. What is the area of the region in the complex plane consisting of all points z satisfying both $\left|\frac{1}{z} - 1\right| < 1$ and $|z-1| < 1$? ($|z|$ denotes the magnitude of a complex number, i.e. $|a+bi| = \sqrt{a^2+b^2}$.)

   Answer: $\mathbf{\frac{2\pi}{3} + \frac{\sqrt{3}}{4}}$

   Solution: Let $z = a + bi$. The first inequality becomes $\left|\frac{1}{a+bi} - 1\right| < 1$, which we can write as $\left|\frac{a-bi}{a^2+b^2} - \frac{a^2+b^2}{a^2+b^2}\right| < 1$. Using the definition of magnitude, we have $\frac{(a-(a^2+b^2))^2+b^2}{(a^2+b^2)^2} < 1$, which can be expanded and factored to give

$$\begin{aligned}
&\frac{a^2 - 2a(a^2+b^2) + (a^2+b^2)^2 + b^2}{(a^2+b^2)^2} < 1 \\
\Rightarrow\ &\frac{1 - 2a + a^2 + b^2}{a^2+b^2} < 1 \\
\Rightarrow\ &\frac{1-2a}{a^2+b^2} < 0.
\end{aligned}$$

   Thus, the first inequality is satisfied when $a > \frac{1}{2}$. For the second inequality, we see that the points are the interior of the circle centered at 1 with radius 1. We now need to find the area of the region of the circle to the right of $a = \frac{1}{2}$. Let the center of the circle be O, and let $a = \frac{1}{2}$ intersect the circle at points A and B. Let the midpoint of AB be M. We see that $\triangle AOM$ is a 30-60-90 triangle, so $\angle AOB = 120^\circ$. Thus, the area we want is $\frac{2}{3}$ of the area of the circle plus the area of $\triangle AOB$, giving us $\boxed{\frac{2\pi}{3} + \frac{\sqrt{3}}{4}}$.

3. Determine

$$\left\lfloor \prod_{n=2}^{2022} \frac{2n+2}{2n+1} \right\rfloor,$$

   given that the answer is relatively prime to 2022.

   Answer: 29

   Solution: Let this product have value P. The desired answer is $\lfloor P \rfloor$. We would like to find a way to make this product telescope. Consider

$$\frac{(2n+2)(2n+3)}{(2n+1)(2n+2)}$$

Note that

$$\frac{(2n+2)(2n+3)}{(2n+1)(2n+2)} < \frac{(2n+2)^2}{(2n+1)^2} \iff \frac{2n+3}{2n+2} < \frac{2n+2}{2n+1}$$

So we can bound this product below by

$$P^2 > \prod_{n=2}^{2022} \frac{(2n+2)(2n+3)}{(2n+1)(2n+2)} > 809 \implies P > 28$$

However, by similar logic, we may also use the fraction $\frac{(2n+2)(2n+1)}{(2n+1)(2n)}$ to bound our product below. The tricky part here is that we must exclude some terms to increase our accuracy slightly. We have

$$P^2 < \frac{6^2}{5^2} \cdot \frac{8^2}{7^2} \prod_{n=3}^{2022} \frac{(2n+2)(2n+1)}{(2n+1)(2n)} = \frac{36}{25} \cdot \frac{64}{49} \cdot \frac{4046}{8} = \frac{36}{25} \cdot \frac{8}{49} \cdot 4046 = \frac{48}{49} \cdot \frac{6}{5} \cdot \frac{4046}{5}$$

$$< \frac{48}{49} \cdot \frac{6}{5} \cdot 810 = \frac{48}{49} \cdot 972 < \frac{48}{49} \cdot 980 = 48 \cdot 20 = 960 < 961$$

$$\implies P < 31$$

Hence, since P is also coprime to 2022, $P \neq 30$, and so $P = \boxed{29}$.