Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise.
No calculators.

1. Compute the slope of the line tangent to $y^2 = x^3 + x + 1$ at the point $(0, 1)$.

2. For how many real numbers x do we have that $\log_5(1 + x) = x$?

3. Eric is standing on the circumference of a circular barn with a radius of 100 meters. Ross starts at the point on the circumference diametrically opposite to Eric and starts moving toward him along the circumference such that the straight-line distance between them decreases at a constant rate of 1 meter per second. When Ross is at an angle of $\frac{\pi}{2}$ from Eric, what is the rate of change of the angle between them, in radians per second? (The angle between Ross and Eric is measured with respect to the center of the circular barn.)

4. The function $f(x, y)$ has value $-\ln(a)$ whenever $x^2 + \frac{y^2}{4} = a^2$ and $0 < a \leq 1$, and 0 otherwise. Compute the volume contained in the region below this function and above the xy-plane.

5. Compute
$$\int \sin(\sin(x)) \sin(2x) \, \mathrm{d}x.$$

6. Compute
$$\int_{-1}^{1} \frac{1 + \cos(x)}{1 + 3^x} \, \mathrm{d}x.$$

7. A stone is bouncing on a pond. It starts at height 1. Each time it bounces on the pond, its height x changes to a uniformly random height between 0 and x. If the height ever drops below $\frac{1}{10}$, the next time it hits the pond it will sink. What is the expected number of times the stone will bounce before sinking (not counting the sinking as a bounce)?

8. Let $r_1(t) \leq r_2(t) \leq r_3(t)$ be the roots of $x^3 + tx + 2$. When $t = -3$, compute $r_1'(t)$.

9. Let $\triangle ABC$ be an equilateral triangle with side length 4. Let P be a point chosen uniformly and at random in the interior of $\triangle ABC$. Determine the probability that a square of side length 1 with a corner at P can be rotated to lie entirely within $\triangle ABC$.

10. Define the double factorial via $(2n-1)!! = (2n-1)(2n-3)\cdots 1$. Compute the unique pair (a, c) with $c > 0$ and $a \in (0, \infty)$ such that
$$\lim_{n\to\infty} \frac{c^n (4n-1)!!}{(2n-1)!!(2n-1)!!} = a.$$