This is set 1.

1. To convert between Fahrenheit, F, and Celsius, C, the formula is $F = \frac{9}{5}C + 32$. Jennifer, having no time to be this precise, instead approximates the temperature of Fahrenheit, $\widehat{F}$, as $\widehat{F} = 2C + 30$. There is a range of temperatures $C_1 \leq C \leq C_2$ such that for any C in this range, $|\widehat{F} - F| \leq 5$. Compute the ordered pair (C_1, C_2).

2. Compute integer x such that $x^{23} = 27368747340080916343$.

3. The number of ways to flip n fair coins such that there are no three heads in a row can be expressed with the recurrence relation

$$S(n+1) = a_0 S(n) + a_1 S(n-1) + ... + a_k S(n-k)$$

for sufficiently large n and k where $S(n)$ is the number of valid sequences of length n. What is

$$\sum_{n=0}^{k} |a_n|?$$

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4. For how many three-digit multiples of 11 in the form $\underline{abc}$ does the quadratic $ax^2 + bx + c$ have real roots?

5. William draws a triangle $\triangle ABC$ with $AB = \sqrt{3}, BC = 1$, and $AC = 2$ on a piece of paper and cuts out $\triangle ABC$. Let the angle bisector of $\angle ABC$ meet AC at point D. He folds $\triangle ABD$ over BD. Denote the new location of point A as A'. After William folds $\triangle A'CD$ over CD, what area of the resulting figure is covered by three layers of paper?

6. Compute $(1)(2)(3) + (2)(3)(4) + \ldots + (18)(19)(20)$.

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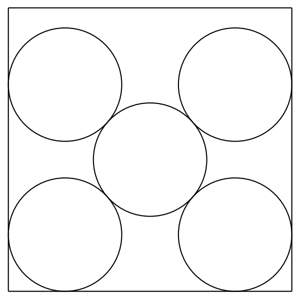
7. An ant starts at the point $(0, 0)$. It travels along the integer lattice, at each lattice point choosing the positive x or y direction with equal probability. If the ant reaches $(20, 23)$, what is the probability it did not pass through $(20, 20)$?

8. Let $a_0 = 2023$ and a_n be the sum of all divisors of a_{n-1} for all $n \geq 1$. Compute the sum of the prime numbers that divide a_3.

9. Five circles of radius one are stored in a box of base length five as in the following diagram. How far above the base of the box are the upper circles touching the sides of the box?



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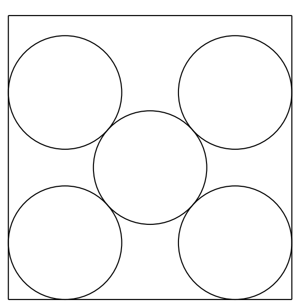

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10. Three rectangles of dimension $X \times 2$ and four rectangles of dimension $Y \times 1$ are the pieces that form a rectangle of area $3XY$ where X and Y are positive, integer values. What is the sum of all possible values of X?

11. Suppose we have a polynomial $p(x) = x^2 + ax + b$ with real coefficients $a + b = 1000$ and $b > 0$. Find the smallest possible value of b such that $p(x)$ has two integer roots.

12. Ten square slips of paper of the same size, numbered $0, 1, 2, ..., 9$, are placed into a bag. Four of these squares are then randomly chosen and placed into a two-by-two grid of squares. What is the probability that the numbers in every pair of blocks sharing a side have an absolute difference no greater than two?

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13. Let $\triangle ABC$ be an equilateral triangle with side length 1. Let the unit circles centered at A, B, and C be Ω_A, Ω_B, and Ω_C, respectively. Then, let Ω_A and Ω_C intersect again at point D, and Ω_B and Ω_C intersect again at point E. Line BD intersects Ω_B at point F where F lies between B and D, and line AE intersects Ω_A at G where G lies between A and E. BD and AE intersect at H. Finally, let CH and FG intersect at I. Compute IH.

14. Suppose Bob randomly fills in a 45×45 grid with the numbers from 1 to 2025, using each number exactly once. For each of the 45 rows, he writes down the largest number in the row. Of these 45 numbers, he writes down the second largest number. The probability that this final number is equal to 2023 can be expressed as $\frac{p}{q}$ where p and q are relatively prime positive integers. Compute the value of p.

15. f is a bijective function from the set $\{0, 1, 2, \cdots, 11\}$ to $\{0, 1, 2, \cdots, 11\}$, with the property that whenever a divides b, $f(a)$ divides $f(b)$. How many such f are there? *A bijective function maps each element in its domain to a distinct element in its range.*

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16. When not writing power rounds, Eric likes to climb trees. The strength in his arms as a function of time is $s(t) = t^3 - 3t^2$. His climbing velocity as a function of the strength in his arms is $v(s) = s^5 + 9s^4 + 19s^3 - 9s^2 - 20s$. At how many (possibly negative) points in time is Eric stationary?

17. Consider a triangle $\triangle ABC$ with angles $\angle ACB = 60°$, $\angle ABC = 45°$. The circumcircle around $\triangle ABH$, where H is the orthocenter of $\triangle ABC$, intersects BC for a second time in point P, and the center of that circumcircle is O_c. The line PH intersects AC in point Q, and N is center of the circumcircle around $\triangle AQP$. Find $\angle NO_cP$.

18. If x, y are positive real numbers and $xy^3 = \frac{16}{9}$, what is the minimum possible value of $3x + y$?

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19. $A_1A_2...A_{12}$ is a regular dodecagon with side length 1 and center at point O. What is the area of the region covered by circles $(A_1A_2O), (A_3A_4O), (A_5A_6O), (A_7A_8O), (A_9A_{10}O)$, and $(A_{11}A_{12}O)$? (ABC) *denotes the circle passing through points* $A, B,$ *and* C.

20. Let $N = 2000\ldots0x0\ldots00023$ be a 2023-digit number where the x is the 23rd digit from the right. If N is divisible by 13, compute x.

21. Alice and Bob each visit the dining hall to get a grilled cheese at a uniformly random time between 12PM and 1PM (their arrival times are independent) and, after arrival, will wait there for a uniformly random amount of time between 0 and 30 minutes. What is the probability that they will meet?

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22. Consider the series $\{A_n\}_{n=0}^{\infty}$, where $A_0 = 1$ and for every $n > 0$,

$$A_n = A_{\left[\frac{n}{2023}\right]} + A_{\left[\frac{n}{2023^2}\right]} + A_{\left[\frac{n}{2023^3}\right]},$$

where $[x]$ denotes the largest integer value smaller than or equal to x. Find the $(2023^{3^2} + 20)$-th element of the series.

23. The side lengths of triangle $\triangle ABC$ are 5, 7 and 8. Construct equilateral triangles $\triangle A_1BC$, $\triangle B_1CA$, and $\triangle C_1AB$ such that A_1, B_1, C_1 lie outside of $\triangle ABC$. Let A_2, B_2, and C_2 be the centers of $\triangle A_1BC$, $\triangle B_1CA$, and $\triangle C_1AB$, respectively. What is the area of $\triangle A_2B_2C_2$?

24. There are 20 people participating in a random tag game around an 20-gon. Whenever two people end up at the same vertex, if one of them is a tagger then the other also becomes a tagger. A round consists of everyone moving to a random vertex on the 20-gon (no matter where they were at the beginning). If there are currently 10 taggers, let E be the expected number of untagged people at the end of the next round. If E can be written as $\frac{a}{b}$ for a, b relatively prime positive integers, compute $a + b$.

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This is set 9.

25. You are given that 1000! has 2568 decimal digits. Call a permutation π of length 1000 good if $\pi(2i) > \pi(2i-1)$ for all $1 \leq i \leq 500$ and $\pi(2i) > \pi(2i+1)$ for all $1 \leq i \leq 499$. Let N be the number of good permutations. Estimate D, the number of decimal digits in N.

    You will get $\max\left(0, 25 - \left\lceil \frac{|D-X|}{10} \right\rceil\right)$ points, where X is the true answer.

26. A year is said to be *interesting* if it is the product of 3, not necessarily distinct, primes (for example $2^2 \cdot 5$ is interesting, but $2^2 \cdot 3 \cdot 5$ is not). How many interesting years are there between 5000 and 10000, inclusive?

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27. Sam chooses 1000 random lattice points (x, y) with $1 \leq x, y \leq 1000$ such that all pairs (x, y) are distinct. Let N be the expected size of the maximum collinear set among them. Estimate $\lfloor 100N \rfloor$. Let S be the answer you provide and X be the true value of $\lfloor 100N \rfloor$.

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